Common tests are linear models

See worked examples and more details at the accompanying notebook: <u>https://lindeloev.github.io/tests-as-linear</u>

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		Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	lcon
egression: $Im(y \sim 1 + x)$	y is independent of x P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	lm(y ~ 1) lm(signed_rank(y) ~ 1)	✓ for N >14	One number (intercept, i.e., the mean) predicts y . - (Same, but it predicts the <i>signed rank</i> of y .)	;; <mark>;</mark>
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y ₁ , y ₂ , paired=TRUE) wilcox.test(y ₁ , y ₂ , paired=TRUE)	$lm(y_2 - y_1 \sim 1)$ lm(signed_rank(y_2 - y_1) ~ 1)	√ f <u>or N >14</u>	One intercept predicts the pairwise y ₂ - y ₁ differences. - (Same, but it predicts the <i>signed rank</i> of y ₂ - y ₁ .)	
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	Im(y ~ 1 + x) Im(rank(y) ~ 1 + rank(x))	√ <u>for N >10</u>	One intercept plus x multiplied by a number (slope) predicts y . - (Same, but with <i>ranked</i> x and y)	- And Marker
Simple r	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y ₁ , y ₂ , var.equal=TRUE) t.test(y ₁ , y ₂ , var.equal=FALSE) wilcox.test(y ₁ , y ₂)	$Im(y \sim 1 + G_2)^A$ $gls(y \sim 1 + G_2, weights=^B)^A$ $Im(signed_rank(y) \sim 1 + G_2)^A$	✓ ✓ for N >11	An intercept for group 1 (plus a difference if group 2) predicts y . - (Same, but with one variance <i>per group</i> instead of one common.) - (Same, but it predicts the <i>signed rank</i> of y .)	¥.
Itiple regression: $Im(y \sim 1 + x_1 + x_2 +)$	P: One-way ANOVA N: Kruskall-Wallis	aov(y ~ group) kruskal.test(y ~ group)	$Im(y \sim 1 + G_2 + G_3 + + G_N)^4$ Im(rank(y) ~ 1 + G_2 + G_3 + + G_N)^4	√ <u>for N >11</u>	An intercept for group 1 (plus a difference if group ≠ 1) predicts y . - (Same, but it predicts the <i>signed rank</i> of y .)	₩Ţ Ţ
	P: One-way ANCOVA	aov(y ~ group + x)	$Im(y \sim 1 + G_2 + G_3 + + G_N + x)^4$	~	- (Same, but plus a slope on x.) Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.	
	P: Two-way ANOVA	aov(y ~ group * sex)	$\begin{array}{l} Im(y \sim 1 + G_2 + G_3 + + G_N + \\ S_2 + S_3 + + S_K + \\ G_2^*S_2 + G_3^*S_3 + + G_N^*S_K) \end{array}$	√	Interaction term: changing sex changes the y ~ group parameters. Note: $G_{2 to N}$ is an <u>indicator (0 or 1)</u> for each non-intercept levels of the group variable. Similarly for $S_{2 to K}$ for sex. The first line (with G _i) is main effect of group, the second (with S _i) for sex and the third is the group × sex interaction. For two levels (e.g. male/female), line 2 would just be "S ₂ " and line 3 would be S ₂ multiplied with each G _i .	[Coming]
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	$\begin{array}{l} \mbox{Equivalent log-linear model} \\ glm(y \sim 1 + G_2 + G_3 + + G_N + \\ S_2 + S_3 + + S_K + \\ G_2^*S_2 + G_3^*S_3 + + G_N^*S_K, \mbox{ family=})^4 \end{array}$	*	Interaction term: (Same as Two-way ANOVA.) Run glm using the following arguments: $glm (model, family=poisson())$ As linear-model, the Chi-square test is $log(y_i) = log(N) + log(\alpha_i) + log(\alpha_j) + log(\alpha_i\beta_j)$ where α_i and β_i are proportions. See more info in the accompanying notebook.	Same as Two-way ANOVA
Mu	N: Goodness of fit	chisq.test(y)	$glm(y \sim 1 + G_2 + G_3 + + G_N,)^A$	~	(Same as One-way ANOVA.)	1W-ANOVA

List of parametric (P) non-parametric (N) tests and equivalent linear models. The notation $y \sim 1 + x$ is R shorthand for $y = 1 \cdot b + a \cdot x$ which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they all are! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is signed_rank = function(x) sign(x) * rank(abs(x)). The variables G_i and S_i are <u>"dummy coded" indocator</u> variables (either 0 or 1) exploiting the fact that when $\Delta x = 1$ between categories the difference equals the slope. Subscripts (e.g., G₂ or y₁) indicate different columns in data. Im requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <u>https://lindeloev.github.io/tests-as-linear</u>.

^A See the note to the two-way ANOVA for explanation of the notation.

^B Same model, but with one variance per group: gls(value ~ 1 + G₂, weights = varIdent(form = ~1|group), method="ML").



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